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Review of "Symmetry and Geometry of the Face on Mars Revealed" by Mark Carlotto

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Here I review certain aspects of the paper "Symmetry and Geometry of the Face on Mars Revealed" written by Mark Carlotto. One of the aims of this review is to determine, using methods of analytical geometry, how many of the elementary symmetries that Carlotto's measurement indicate are independent. Furthermore, I wish to show to what extent these symmetries, if exact, are actually compatible with one another. My comments are to taken in the context of Figures 3-4 through 3-8 of his paper.

In his paper a set of axes are used to define the horizontal and vertical i, j location of each portion of the image. Then a quantity that measures the symmetry in the horizontal and vertical directions is computed as a function of i and j separately. He uses this to define the two axes of symmetry of the feature and their intersection point as the origin. Given these definitions the Face on Mars (FOM) displays a number of interesting and remarkable features (1-19 below). Of those, 12 are independent and are numerically highly compatible. (I list with Roman numerals those features that are arguably intentional and not "merely" deductional or logical consequences of the others.)

1. The axes of symmetry defined by β are symmetrically placed with respect the contour of the Face. (I)
2. The origin coincides with a circular feature **a** on the Face. (II)
3. A rectangle (**EFGH**) centered about this origin has edges that are for the most part parallel to the boundaries of the Face and very nearly has the height to width ratio of 4/3. (III)
4. Circular features **b**, **c**, **d** lie along the axis of symmetry. (IV,V,VI).
5. An ellipse **e1** with eccentricity equal to 0.22 is constructed from the lower left and bottom edge of the Face.
6. Top of ellipse so defined passes through point **c**. (VII)
7. The ellipse **e2** is defined so that its sides are parallel to those of rectangle

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8. An inner rectangle (**ABCD**) has vertices defined by the intersection of **e2** with the two diagonals of the outer rectangle. This has the following two consequences i) It has the same proportions as **EFGH** and ii) its area is 1/2 that of the larger rectangle. Note that these two properties are not independent.
9. The inner rectangle **ABCD** has its two vertical edges defined by the beveled edge of the left side of the FOM and nearly the right side. (VIII)
10. Ellipse **e3** is defined.
11. The ellipse **e3** intersects point **b**. (IX)
12. The area of the ellipse **e3** is very nearly equal to area of **e1**. (X)
13. Ellipse **e4** is defined.
14. Its aspect ratio is 4/3 (XI). This is perhaps the most remarkable of the features in my opinion. It is not a consequence of the earlier symmetries but rather an independent and (because of the repetition of the ratio) arguably intentional feature.
15. The distance from the top of **e4** to the horizontal axes to the distance from bottom of **e4** is 4/3. This figure becomes exact if the eccentricity is as large as 0.244. However, it is deductional rather than independent.
16. Along the upper left diagonal **e1** intersects the line halfway between the point where the ellipse **e3** intersects the line and the center. This figure (the 1/2) becomes exact if the eccentricity is as large as 0.247... But as with the above it is a deductional rather than an independent or additional and arguably intentional aspect.
17. This intersection occurs near the western eye. (X)
18. Distance **ac** = **dc** (XII). This feature is independent, a new aspect not a logical consequence of the other features.
19. Circle defined by bottom of **ABCD** and vertices **A**, **B** passes through **d**. This feature can be logically inferred from 18) and earlier symmetries. It is exactly consistent with all of the above (save 15 and 16) if $\varepsilon = 0.248...$

Note that all three determined values (in 15,16,19) of the eccentricity of this ellipse are within roughly one tenth of 1% of one another. Given this outline above, below I present the details.

His symmetry position clearly occurs at the center of a circular feature **a**. (We call this property I1 - properties that are arguably intentional I designate with the letter I.) There are three other nearly circular features that are along this center vertical line of symmetry. Call these properties I2, I3, and I4. Using some curve fitting method (not specified) Carlotto finds that the southwestern and western edges of the base of the FOM platform conforms to a portion of an ellipse with $\varepsilon = 0.22$. The fit is such that the southwestern and western portions of the ellipse are parallel to the corresponding portions of the FOM. An outcome is that the eccentricity ε lies between 0 and 1, $0 < \varepsilon < 1$ so that it is an ellipse and not a hyperbola. The fit is such that one of the axes of the ellipse is the vertical axis defined by β. The semimajor axis length of the ellipse (call it a_1) is also fixed by the fit as is the location along the vertical axis of lower focus of the ellipse. The equation of this ellipse is

$$\left(\frac{x}{a_1}\right)^2 + \left(\frac{y - y_1}{b_1}\right)^2 = 1 \tag{1}$$

where b_1 is the semiminor axes of the ellipse and y_1 is the vertical location of the center of the ellipse in the coordinate system defined by β. The semimajor and semiminor axes of the ellipse are related to the eccentricity by

$$\left(\frac{b_1}{a_1}\right)^2 = 1 - \varepsilon^2 \tag{2}$$

This ellipse has four parameters ($x_1 = 0, y_1, b_1, a_1$) that are fixed by this fit. The coordinates x, y are measured relative to center the circular feature **a**. The first aspect of this ellipse that might be called intentional is the fact that it intersects the central circular feature **c**. (Call this property I5). At the intersection point $x = 0$ so that the y coordinate of **c** is

$$y_c = y_1 + b_1 \tag{3}$$

while the vertical coordinate of the bottom of the ellipse and FOM platform is y_1 - b_1. Note the fact that this same ellipse on its eastern side nearly fits the southeastern and eastern edges. This is weakly implied by the fact that the center of symmetry is approximately the geometrical center of the mesa. We do not therefore assign an intentionality factor to this. One can construct a rectangle with sides parallel to the bottom and western side of the ellipse and with a vertex at **A**. The proportions of this ellipse are quite close to 3/4. Although it is difficult to assign a probability to this proportion since it is just one of many simple proportions that one might consider remarkable, it closeness to this fraction may be considered "intentional". (Other simple fractions that might be considered intentional would be 1, 1/2, 1/3, 2/3, 1/√2,...) So we call this property I6. If one mirrors this ellipse to the top of the head then the north edge of the FOM is very nearly parallel to the top edge of the rectangle. Again this is weakly implied by the fact that the center of symmetry is approximately the geometrical center of the mesa and therefore we do not assign an addition "intentionality" factor to this. One can also construct rectangles that mirror the two outlining the western portion of the FOM. Let **EFGH** be the union of the four rectangles. Take its four corners to be at

$$x = \pm 3\ , \quad y = \pm 4. \tag{4}$$

Its area, in units in which the width is 6 units and the height is 8 units, is 48 square units. The ellipse **e2** inscribed within its boundaries would automatically have an aspect ratio of 3/4. The same could be said about the rectangle **ABCD** whose corners are circumscribed by **e2**. Its proportions are the same as that of the outer rectangle. As a consequence the area of the inner rectangle is 1/2 that of the outer rectangle. This means that its height is the height of the outer rectangle divided by √2 while its width is that of the outer rectangle divided by √2. To see that fact is a logical consequence of earlier features and not new note that the ellipse **e2** is described by the formula

$$\frac{x^2}{3^2} + \frac{y^2}{4^2} = 1 \tag{5}$$

while the straight lines that form the diagonals have the equations

$$\begin{aligned} y &= \frac{4}{3}x \\ y &= -\frac{4}{3}x. \end{aligned} \tag{6}$$

If we substitute these into the ellipse equation we obtain the x and y coordinates of the intersection of the diagonals with the ellipse. The four corners corresponding to these intersection points are

$$x = \pm\frac{3}{\sqrt{2}}\ , \quad y = \pm\frac{4}{\sqrt{2}} \tag{7}$$

and thus the area of the rectangle would be 24 units. Since this logically follows from the first, we do not assign an intentionality factor. However, the fact is that the left edge of the inner rectangle does parallel the beveled edge of the western side of the inner portion of the FOM mesa. We call this property I7. The corresponding eastern edge of the rectangle only crudely demarcates the edge eastern side of the inner portion of the FOM mesa which comes in toward the edge of the rectangle and then out again. Carlotto constructs an ellipse **e3** from a fit to the northwestern edge of the FOM platform. Its equation is

$$\left(\frac{x}{a_3}\right)^2 + \left(\frac{y - y_3}{b_3}\right)^2 = 1. \tag{8}$$

This ellipse has four parameters ($x_3 = 0, y_3, b_3, a_3$) that are fixed by this fit. We call I8 the property that this ellipse passes through circular feature **b** that lies along the lateral axis of symmetry. This means that its y coordinate is

$$y_b = y_3 - b_3. \tag{9}$$

Carlotto states that the area of this ellipse is very nearly the same as that of **e1**. Using the fact that the area of an ellipse with semimajor and semiminor axes a and b is ab we see that this would imply that

$$\pi a_1 b_1 = \pi a_3 b_3. \tag{10}$$

Since the parameters a_3, b_3 have no other connection to those of a_1, b_1 we conclude that this near area equality is an independent and arguably intentional property. Call it I9. Note, however, that the western part of the ellipse **e3** lies to the west of the western edge of the base of the FOM while its eastern part lies just at the eastern edge of the base of the FOM.

Carlotto then constructs the ellipse **e4** within **e3** that passes through the circular features **b** and **c** and is tangent to **ABCD**. Its equation is

$$\left(\frac{x}{a_4}\right)^2 + \left(\frac{y - y_4}{b_4}\right)^2 = 1. \tag{11}$$

His measurements find that

$$\frac{b_4}{a_4} = \frac{3}{4}. \tag{12}$$

Is this simple ratio an independent feature or is it a logical consequence of the earlier ones? Since the outer rectangle has proportions 3/4 and the ellipse **e1** lies just within the boundary of the platform we take $a_1 = 3$ units of length. (If $\varepsilon = 0.22$ then in these units $b_1 = 2.93$.) The height of the FOM (from bottom to top) is then 8 units of length. This is the same as the height of the outer rectangle **EFGH**. Its width is 6 units. The width of the inner rectangle is thus $6/\sqrt{2}$ units and hence $a_4 = 3/\sqrt{2}$ units. Now for the ellipse **e4** we must have

$$2b_4 = y_c - y_b \tag{13}$$

and furthermore that

$$y_c + 4 = 2b_1$$
$$4 - y_b = 2b_3. \tag{14}$$

Thus adding these two equations we have

$$2b_4 = 2b_1 + 2b_3 - 8 \tag{15}$$

Now we have that the equality of the areas of **e1** and **e3** means

$$a_3 b_3 = b_1 a_1 = 3b_1 \tag{16}$$

so that

$$b_3 = \frac{3b_1}{a_3} \tag{17}$$

and

$$b_4 = b_1 + \frac{3b_1}{a_3} - 4 \tag{18}$$

But since a_3 is not known as a logical consequence of the earlier relations we cannot logically determine b_4 nor therefore the ratio b_4/a_4. That is, given the relations above and b_1, the quantity a_3 could be "intentionally" manipulated to give the desired b_4. Hence we conclude that the property $b_4/a_4 = 3/4$ is independent and the fact that it repeats the ratio found earlier for the rectangles is particularly significant. We call this very important property I10. Note that with $b_4 = 3/4\ a_4$ this will then determine a_3 by the equation

$$a_3 = \frac{3b_1}{b_4 + 4 - b_1} = \frac{3b_1}{9/(4\sqrt{2}) + 4 - b_1}$$
$$b_3 = 9/(4\sqrt{2}) + 4 - b_1. \tag{19}$$

Both are expressed in terms of b_1 which in turn is related to the eccentricity. The next property that Carlotto finds is that the distance from the bottom of **e4** to the horizontal axes to the distance from top of **e4** is 3/4. Let us see if this is implied by the earlier measurements.

The distance from bottom of **e4** to the horizontal axes is -y_b while the distance from top of **e4** is y_c. From our earlier work

$$-\frac{y_b}{y_c} = \frac{b_3 - 2}{b_1 - 2} = \frac{9/(4\sqrt{2}) + 2 - b_1}{b_1 - 2}. \tag{20}$$

If we assume that this ratio is 3/4 then we can solve this for b_1 and thus ε .

$$\begin{aligned} &\frac{3}{4} = \frac{9/(4\sqrt{2}) + 2 - b_1}{b_1 - 2} \quad \text{or} \\ &b_1 = 2 + \frac{9}{7\sqrt{2}} = \sqrt{1-\varepsilon^2}a_1 = 3\sqrt{1-\varepsilon^2} \text{ or} \\ &\varepsilon = \sqrt{1-x^2}; \quad x = \frac{2}{3} + \frac{3}{7\sqrt{2}} = .9697. \end{aligned} \tag{21}$$

This leads to ε = 0.244... compared with Carlotto's measured value of 0.22. This latter value of ε would lead to an aspect ratio of about 0.71. Should the eccentricity be as large as 0.244 then his 3/4 figure could be exact. In either event, the appearance of this ratio is not so much intentional as deductional (i.e. following from earlier proportions).

The next property that Carlotto notices is that the ellipse **e1** intersects the upper left diagonal halfway between the point where ellipse **e3** intersects it and the center. Because of the simple ratio this would be considered intentional if it is independent. In order to see if it is independent, note that the equation of the upper left diagonal is

$$y = -\frac{4}{3}x \tag{22}$$

The coordinates of the **e1** intersection are $(x_{1l}, y_{1l}) = x_{1l}\,(1, -4/3)$ and since

$$\left(\frac{x_{1l}}{a_1}\right)^2 + \left(\frac{-\frac{4}{3}x_{1l} - y_1}{b_1}\right)^2 = 1 = \left(\frac{x_{1l}}{3}\right)^2 + \left(\frac{-\frac{4}{3}x_{1l} - y_1}{b_1}\right)^2 \tag{23}$$

we obtain

$$x_{1l} = \frac{-\frac{4y_1}{3b_1^2} - \sqrt{\frac{1}{9} - \frac{y_1^2}{b_1^2 9} + \frac{16}{9b_1^2}}}{\frac{1}{9} + \frac{16}{9b_1^2}} \tag{24}$$

and similarly

$$x_{3l} = \frac{-\frac{4y_3}{3b_3^2} - \sqrt{\frac{1}{a_3^2} - \frac{y_3^2}{b_3^2 a_3^2} + \frac{16}{9b_3^2}}}{\frac{1}{a_3^2} + \frac{16}{9b_3^2}}. \tag{25}$$

Now from our earlier work we know a_1, b_1, a_3, and b_3. We also know y_b and y_c. Hence we can determine y_3 and y_1. This implies that we can determine x_{3l} and x_{1l} and hence the ratio of lengths along the diagonal would be fixed and not independent. Carlotto's measurement is equivalent to $x_{1l} = 0.5x_{3l}$. Assuming all the other simple integer ratios involving 3/4 are exact, the only imprecise quantity is the eccentricity or b_1. From our analysis above

$$\begin{aligned} y_1 &= y_c - b_1 = b_1 - 4 = -1.07 \\ y_3 &= y_b + b_3 = 4 - b_3 = 1.34 \end{aligned} \tag{26}$$

We find that the relation $x_{1l}/x_{3l} = 1/2$ could be exact if the eccentricity of **e1** were allowed to increase to 0.247. (The algebra cannot be solved analytically for b_1.) Using $\varepsilon = 0.22$ we would obtain

$$\frac{x_{1l}}{x_{3l}} = .506. \tag{27}$$

In either event, this property would not be intentional but would be instead a logical consequence. An intentional feature that is noted is the fact the **e1** intersection is very near the eye feature. We refer to this property as I11. The next feature he measures is that the distance **a** - **c** is equal to the distance **d** - **c**. This would be an intentional feature since no previous geometrical feature has been related to the position of **d** other than it be on the vertical line of symmetry. Thus we call this property I12.

The last simple geometrical feature that Carlotto measures is that one can draw a circle that touches the bottom of **ABCD** and its top two vertices **A** and **B** and at the same time passes through **d**. Again we need to ask the question whether this is an independent and arguably intentional feature. The diameter of the circle is equal to the height of the inner rectangle plus the distance from the top of the inner rectangle to feature **d**. Now the two right triangles that combine to make the isosceles triangle defined by the points **A**, **B**, and the center point between **C** and **D** appear (scaled down) back to back in the isosceles triangle defined by **A**, **B**, and the point **d** on the circle. Geometry will thus allow us to determine the radius of the circle therefore and if it is the same as the radius of the circle obtained by knowing the position of **d** through the fact that **a** - **c** is equal to the distance **d** - **c** then this will be a logically implied feature and not an independent intentional feature. Using the properties of the 3-4-5 triangles and the above geometrical feature, the diameter of the circle is

$$D_o = \frac{8}{\sqrt{2}} + \frac{3}{8}\frac{3}{\sqrt{2}} = \frac{73}{8\sqrt{2}} \tag{28}$$

with the first number on the left hand side being the height of the inner rectangle while the second is the factor 3/8 times the half width of the inner rectangle (i.e. the height above of the top of the circle above the top of the rectangle). Now we see if we can compute this same diameter using the information above about the equality of the distance **a** - **c** and the distance **d** - **c**. The second way of computing the diameter is the half height of the inner rectangle plus twice the **a** - **c** distance. This is

$$D_o = \frac{4}{\sqrt{2}} + 2y_c = \frac{4}{\sqrt{2}} + 4b_1 - 8. \tag{29}$$

Equating this to the above value of D_o would give

$$b_1 = 2 + \frac{41}{32\sqrt{2}} = 2.906 \tag{30}$$

which is in reasonable agreement with the figure of 2.93 corresponding to ε = 0.22. So, this is a derived feature and not an "intentional" one. However, we can use it to derive an exact expression for the eccentricity if we equate the two values exactly. Then we obtain

$$\begin{aligned} \varepsilon &= \sqrt{1 - \left(\frac{b_1}{a_1}\right)^2} \\ &= \sqrt{1 - \left(\frac{2}{3} + \frac{41}{96\sqrt{2}}\right)^2} = .248... \end{aligned} \tag{31}$$

So, even though this is extremely close to the value of 0.247... they are not exactly the same.

In summary, the various measurements of ratios that Carlotto made, if taken at face value, would be consistent to a high degree of accuracy. That is, three of the measurements (15,16,19) involve simple ratios that can be made consistent with all the other ones for three different values of ε very close to each other (0.244...,0.247...,0.248...). In my opinion, the consistency of these values together with the symmetry properties I1 - I12 argue in favor of an artificial origin of numerous aspects of the FOM mesa. The early Viking images of the FOM feature showed it to have virtually universally recognizable face-like features. It is of interest that just as the erosion of some of those features becomes more apparent at higher resolution, unmistakable geometrical aspects not discernible in the earlier Viking images emerge in this latest high resolution image of the Face on Mars.

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